Elasticity and market equilibrium

Consider a market with 800 consumers. Each one has a Marshallian demand for the good y given by $y = m/p_y$, where m is the income.

1. Calculate the aggregate demand y^A
2. Calculate the price elasticity of aggregate demand. And show that this elasticity is equal to applying the natural logarithm to the aggregate demand and then deriving with respect to the natural logarithm of the price $\frac{\partial ln(y^A)}{\partial ln(p_y)}$, (this is another way to calculate elasticity).
3. Assume there is only one firm in the market that has the following long-term cost function: $C(y) = 150 + y^2$. However, this firm is state-owned and acts as if it were in a perfect competition market (matching price to marginal cost). Find the equilibrium quantity and price.
4. Now assume that the firm behaves as if it were a monopoly but changes the demand it faces to the following inverse demand $p = 50-y$, maximize its profits and obtain the equilibrium price and quantity.
5. If the demand faced by the firm were more inelastic, would this benefit the monopolist, harm them, or be indifferent to them? Justify conceptually or with an example.

Solution

1. If each individual has the following demand m/p_y. Multiplying by 800 we have the aggregate demand:

$$y^A = \frac{800M}{p_y}$$

2. We calculate the elasticity with the formula:

$$\frac{Ey^A}{Ep_y} = \frac{\partial y^A}{\partial p_y}\frac{p_y}{y^A} = -\frac{m800}{p_y^2}\frac{p_y}{800m/p_y} = -1$$

Using the other form $ln(y^A) = ln(800M) - ln(p_y)$:

$$\frac{\partial ln(y^A)}{\partial ln(p_y)} = -1$$

3. We obtain the supply curve which is equal to the marginal cost

$$C'_y = 2y$$

We equate with the inverse demand

$$800M/y = 2y$$
$$400M = y^2$$
$$y = 20\sqrt{M}$$

We obtain the price:

$$p_y = 800M/(20\sqrt{M}) = 40\sqrt{M}$$

4.

$$B = (50 - y)y - 150 - y^2$$

We calculate the first order condition:

$$50 - 2y - 2y = 0$$
$$12.5 = y$$

$$p = 50 - 12.5 = 37.5$$

5. A more inelastic demand would benefit the firm as it would allow it to charge a higher price with a smaller quantity. This can be shown with an example. The previous profits were:

$$B = 12.5 * 37.5 - [150 - 12.5^2] = 475$$

The demand elasticity is:

$$\frac{\partial Y}{\partial P}\frac{P}{Y} = \frac{-1}{50 - P}(P)$$

If we change the demand to a more inelastic one by changing the slope of the demand to a lesser one.

$$Y = 50 - 0.5P$$

Maximizing again:

$$B = (100 - 2y)y - 150 - y^2$$
$$B'y = 50 - 4y - 2y = 0$$
$$y = 50/6 = 8.33$$

And the price would be:

$$p = 100 - 2 * 8.33 = 83.34$$

Now the profits are:

$$B = 8.33 * 83.34 - (150 - (8.33)^2) = 613.61$$

Which is higher than before